IV. Conclusion

In conclusion, a general expression for the gain in terms of S-parameters, based on the physical concept of the input and output impedances of the self-terminated circuit, has been provided. This expression is mathematically equivalent to the one provided in terms of T-parameters in the above paper, and invalidates the S-parameter formulation provided by Randall and Hock.

Acknowledgment

The author wishes to thank H. S. Sodhim, Verticom Inc., Santa Rosa, CA, for useful discussions and for reviewing this paper's manuscript.

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Authors' Reply

Mitch Randall and Terry Hock

Cascio's comments and S-parameter derivation touch on an important fundamental concept underlying the derivation described in the above paper.[1] The above paper did not go into depth explaining some of the subtleties related to this, but we welcome the opportunity to do so now.

In the above paper, we consider an infinite series of identical networks with the goal of accounting for the impedance mismatch when a single network is self-connected. However, care has to be taken to interpret and use this concept appropriately since an infinite series of networks is *not* the same as a single self-connected network.

In particular, consider a well-designed single-ended open-loop oscillator network Z. To this network we add a resistor R to a new single-ended ground defining a new impedance matrix $\tilde{Z}$, as shown in Fig. 1, so that

$$\tilde{Z} = \begin{bmatrix} z_i + R & z_r + R \\ z_f + R & z_o + R \end{bmatrix}. \quad (1)$$

It is self evident that we have changed nothing fundamental about the operation of the oscillator when self-connected; its operation frequency, loop gain, startup time, loaded Q, etc, remain unchanged. When self-connected, no current flows through the added resistance.

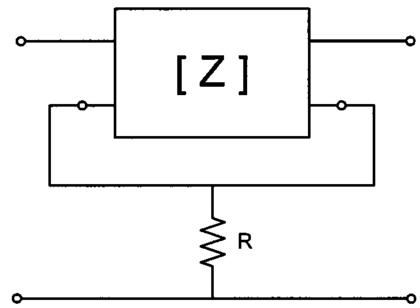

Fig. 1. Resistor R added to an oscillator network as shown changes nothing about its closed-loop performance. The expression characterizing oscillator performance should not depend on this resistor.

However, we *have* changed the open-loop two-port network and its response to various steady-state test signals.

When an infinite series of the new networks $\tilde{Z}$ are cascaded and a steady-state test signal is applied, current will, in general, flow through the resistance R. For example, as the resistance R increases to very large values $(R \gg \tilde{z}_i)$, the new network input impedance approaches $\tilde{z}_i = R$ and the "gain" $\tilde{G}$ approaches unity, as can be shown from the Z-parameter expression

$$\lim_{R\to\infty} \tilde{g} = \frac{\tilde{z}_f}{\tilde{z}_i + \tilde{z}_o} = \frac{z_f + R}{z_i + z_o + 2R} = \frac{1}{2}. \quad (2)$$

Thus,

$$\begin{aligned} \lim_{R\to\infty} \tilde{G} &= \lim_{R\to\infty} \frac{2\tilde{g}}{1 + \sqrt{1 - 4\frac{\tilde{z}_r}{\tilde{z}_f}\tilde{g}^2}} \\ &= \lim_{R\to\infty} \frac{2\left(\frac{1}{2}\right)}{1 + \sqrt{1 - 4\frac{z_r + R}{z_f + R}\left(\frac{1}{2}\right)^2}} \\ &= 1. \end{aligned} \quad (3)$$

This does not invalidate the approach since, when each network is initialized with identical exponentially growing sine waves at the frequency of oscillation, no current flows through the resistors. What is needed is a trick to remove the effect of the added resistor in our *steady-state* analysis while preserving the portion of the impedances that affect the oscillations. To do this, we consider a general single-ended open-loop oscillator network Z. The network can be represented using an ideal unilateral voltage amplifier, as shown in Fig. 2. When the circuit is drawn in this way, it is easy to see that the element z_r plays no role in the oscillations and, therefore, can be removed from consideration so that the steady-state gain takes on an unambiguous meaning. This step is described in the above paper and removes not only the added resistance R inserted above, but also any additional series impedance component already present (but perhaps not obvious) in the original network. No such series impedances, whether added externally or intrinsic to the network, are involved in the oscillations. In order to get meaningful and unambiguous results from a steady-state analysis, all of these elements must be removed. The remaining network containing only those elements that are involved in the oscillations is characterized by

$$Z' = \begin{bmatrix} z_i - z_r & 0 \\ z_f - z_r & z_o - z_r \end{bmatrix}. \quad (4)$$

Manuscript received September 17, 2001. This work was supported by the National Science Foundation.

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Publisher Item Identifier S 0018-9480(02)00737-8.

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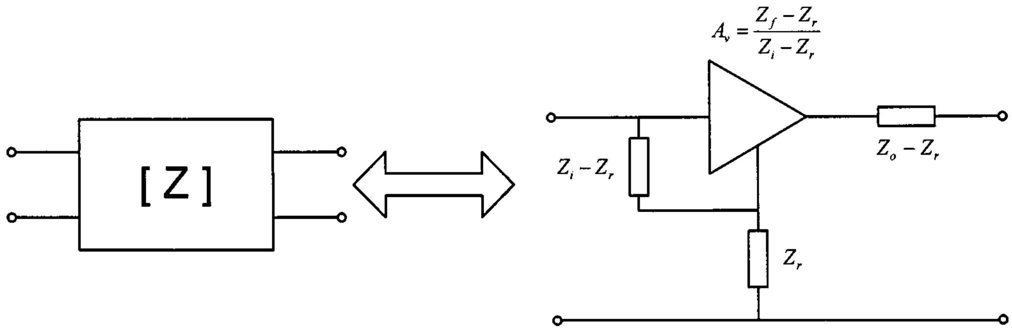

Fig. 2. General network can be redrawn using an ideal voltage amplifier. This form makes it clear that the impedance element z_r, like R in Fig. 1, plays no explicit role in the oscillations.

A simplified form for G results from such a matrix with $z_r' = 0$ and, from that, the S-parameter form as presented in the above paper can be derived.

We acknowledge, as Cascio states, that the T- and S-parameter expressions in the above paper give different results. This is deliberate and important, as described above. Our S-parameter expression includes only the essential elements involved in the oscillations of a network while being independent of the elements of the network not involved in the oscillations. Therefore, we maintain that the S-parameter expression in the above paper is more appropriate for oscillator analysis than that derived by Cascio.

Regarding Cascio's comments about the sign of the expression, strictly speaking, we agree with regard to the T-parameter expression, and Cascio's S-parameter derivation. The S-parameter expression for G in the above paper, however, requires one or the other sign to be definitively chosen in advance. This is so the unwanted element z_r can be identified and removed from the steady-state analysis. For our expression, the analog of changing the sign is to make an exchange of variables $S_{11} \leftrightarrow S_{22}$ and $S_{12} \leftrightarrow S_{21}$.

This makes an excellent point regarding the power of the virtual ground technique. As Cascio points out, this exchange must be made for continuity when the direction of the oscillation reverses over some band. If the loop is redrawn appropriately it will reveal an amplifier and a passive resonator in the familiar and obvious configuration. To say that the direction of oscillation reverses is to say that the amplifier changes its direction of amplification. Practically speaking, then, we can say that this situation is extremely improbable. The point here is that, if drawn as a negative resistance oscillator, for example, we may not have been able to make that statement with such confidence or at all. This technique, then, has managed to remove some of the "magic" out of oscillator design and replace it with intuition. For this, we are quite proud to contribute.

Regarding the paper by Harada [1], until this time, we were not aware of this study and we thank Cascio for bringing it to our attention. It is true that Harada's expression reduces to our expression in the special case $S_{12} = 0$. However, in the general case, where $S_{12} \neq 0$, the results of Harada's expression are not independent of the S-parameter test-set normalizing impedance R_0. This can be verified by noting its appearance in the Z-parameter form

$$G_{\mathrm{Harada}} = \frac{S_{21} - S_{12}}{1 - S_{11}S_{22}} = \frac{z_f - z_r}{z_i + z_o - F} \tag{5}$$

where

$$F = \frac{2 z_f z_r}{z_i + z_o + R_0\left[1 + \dfrac{z_i z_o - z_r z_f}{R_0^2}\right]}. \tag{6}$$

The expression for G presented in the above paper does not suffer from this problem. In addition, it does not give ambiguous results as Cascio's expression would. We maintain that our expression correctly describes the essential characteristics of a closed-loop network based on its open-loop S-parameters.

Cascio correctly points out the typographical error appear on page 1095 of the above paper.

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[1] K. Harada, "An S-parameter transmission model approach to VCO analysis," *RF Design*, pp. 32–42, Mar. 1999.